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Simulation of solution of hyperbolic equations with Orlicz right side

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The paper is devoted to construction of model of hyperbolic equation solution with strictly Orlicz random right side and zero initial and boundary conditions which approximate this solution with given reliability and accuracy in uniform norm.

Key words and phrases: hyperbolic equation, simulation, Orlicz random field.

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Introduction

In this paper we consider the first boundary-value problem of mathematical physics for hyperbolic equation with random right side. Such problems are considered in papers [1, 2, 3, 4]. The paper [4] is devoted to simulation of solution of hyperbolic equations with φ-Sub-Gaussian right side. In papers [2, 3] the suficient conditions for the existance of the solution for hyperbolic equations with homogeneous initial and boundary conditions and with Orlicz right side in form of uniformly convergent in probability series are found. In paperi [1] an estimate of supremum distribution of the problem solution is obtained. In this paper we use this results for simulation of this solution.

1 Simulation of solution

Consider the first boundary-value problem of mathematical physics for nonhomogeneuos hyperbolic equation with zero initial and boundary conditions

$$\frac{\partial^2 u}{\partial x^2} - q(x)u - \frac{\partial^2 u}{\partial t^2} = -\xi(x,t),\ x \in [0,\pi],\ t \in [0,T] \tag{1}$$

$$u|_{t=0} = 0,\ \left.\frac{\partial u}{\partial t}\right|_{t=0} = 0, \tag{2}$$

$$u|_{x=0} = 0,\ u|_{x=\pi} = 0, \tag{3}$$

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where $T > 0$ is some constant, $q(x),\ x \in [0, \pi]$ is continuously differentiable function such that $q(x) \geq 0$, $\xi(x,t),\ x \in [0,\pi],\ t \in [0,T]$ is sample continuous with probability one random field.

The coresponding Sturm-Liouville problem is written as follows

$$\frac{d^2X}{dx^2} - qX + \lambda X = 0$$
$$X(0) = X(\pi) = 0$$

Let $X_n(x)$ be the ortonormal eigen functions and λ_n be the correspondingeigenvalues for this problem. We assume that λ_n are ordered in the ascending order. In view of the restrictions imposed on the function q,all eigenvalues are positive and zero is not an eigenvalue [5].

Put $\mu_n = \sqrt{\lambda_n}$, $B(x,y,t,s) = \mathsf{E}\xi(x,t)\xi(y,s),\ (x,y,t,s) \in [0,\pi]^2 \times [0,T]^2$.
Assume that $B(0,y,t,s) = B(\pi,y,t,s) = 0,\ y \in [0,\pi],\ t \in [0,T],\ s \in [0,T]$;
$B(x,0,t,s) = B(x,\pi,t,s) = 0,\ x \in [0,\pi],\ t \in [0,T],\ s \in [0,T]$.

For every fixed pair $(t,s) \in [0,T]^2$ continue function $B(x,y,t,s)$ as function of x,y on all plane $\mathbb{R}^2$ so that it was periodic function with period 2π by x and by y and that $B(-x,y,t,s) = -B(x,y,t,s) = B(x,-y,t,s)$. In consequence of our assumption that continuation is possible.

Denote

$$B_{i,j}(x,y,t,s) = \frac{\partial^{i+j}}{\partial x^i \partial y^j} B(x,y,t,s),\ 0 \leq i,j \leq 2;$$
$$\Delta_{2,\delta_1,\delta_2} B(x,y,t,s)$$
$$= B(x+\delta_1, y+\delta_2, t,s) - B(x+\delta_1, y, t, s) - B(x, y+\delta_2, t, s) + B(x,y,t,s);$$
$$\Delta_{x,\delta} B(x,y,t,s) = B(x+\delta, y, t, s) - B(x,y,t,s);$$
$$\Delta_{y,\delta} B(x,y,t,s) = B(x, y+\delta, t, s) - B(x,y,t,s);$$
$$\widetilde{B}(x,y,t,s) = B(x,y,t,t) - B(x,y,t,s) - B(x,y,s,t) + B(x,y,s,s).$$

Theorem 1 ([2])**.** *Let in (1) $\xi(x,t)$ is centered strong Orlicz sample continuous with probability one random field from space $L_U(\Omega)$, U satisfies the g-condition (that is $\exists z_0 \geq 0$ $\exists K > 0$ $\exists A > 0$ $\forall x \geq z_0$ $\forall y \geq z_0$: $U(x)U(y) \leq AU(Kxy)$). Let $\varphi(\lambda),\ \lambda > 0$ is continuous increasing function , $\varphi(\lambda) > 0$ for all $\lambda > 0$, $\varphi(\lambda) \to \infty,\ \lambda \to \infty$ is such function that $\dfrac{\lambda}{\varphi(\lambda)}$ is increasing for $\lambda > v_0$, where constant $v_0 \geq 0$. Assume that for all $x \in [0,\pi]$, $y \in [0,\pi]$, $t \in [0,T]$, $s \in [0,T]$ the continuous derivative $\frac{\partial^4}{\partial x^2 \partial y^2} B(x,y,t,s)$ is exsist and for some continuous functions $\tau(\delta_1,\delta_2)$, $\delta_1 \geq 0$, $\delta_2 \geq 0$ та $\tau(\delta)$, $\delta \geq 0$ such that $\tau(\delta_1,\delta_2) > 0$ for $\delta_1 > 0$, $\delta_2 > 0$, $\tau(0,\delta_2) = \tau(\delta_1, 0) = 0$, $\tau(\delta_1,\delta_2)$ is monotonously increasing by δ_1 and by δ_2, $\tau(\delta) > 0$ for $\delta > 0$, $\tau(0) = 0$, $\tau(\delta)$ is monotonously increasing, next conditions hold true:*

– *following series converge*

$$\sum_{k=1}^{\infty}\sum_{m=1}^{\infty} \frac{\tau\left(\frac{\pi}{k}, \frac{\pi}{m}\right)}{km} \varphi(k^2)\varphi(m^2) < \infty,\ \sum_{k=1}^{\infty}\sum_{m=1}^{\infty} \frac{\tau\left(\frac{\pi}{k}\right)}{km^2} \varphi(k^2)\varphi(m^2) < \infty,$$
$$\sum_{k=1}^{\infty}\sum_{m=1}^{\infty} \frac{\tau\left(\frac{\pi}{m}\right)}{k^2 m} \varphi(k^2)\varphi(m^2) < \infty,\ \sum_{k=1}^{\infty}\sum_{m=1}^{\infty} \frac{1}{k^2 m^2} \varphi(k^2)\varphi(m^2) < \infty;$$

– *for all* $\varepsilon > 0$

$$\int_0^{\varepsilon} U^{(-1)}\left(\left(\varphi^{(-1)}\left(\frac{1}{v}\right)\right)^2\right) dv < \infty; \tag{4}$$

– *for all* $0 \le i,j \le 1$ *exist constants* $C_{1,i,j} > 0,\ C_{2,i,j} > 0,\ C_{3,i,j} > 0$ *such that*

$$\sup_{\substack{0\le t\le T\\ 0\le s\le T}} \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} |\Delta_{2,\delta_1,\delta_2} B_{2i,2j}(x,y,t,s)|\, dxdy \le C_{1,i,j}\tau(\delta_1,\delta_2),$$

$$\sup_{\substack{0\le t\le T\\ 0\le s\le T}} \int_0^{\pi}\left(\int_{-\pi}^{\pi} |\Delta_{x,\delta} B_{2i,2j}(x,y,t,s)|\, dx\right) dy \le C_{2,i,j}\tau(\delta),$$

$$\sup_{\substack{0\le t\le T\\ 0\le s\le T}} \int_0^{\pi}\left(\int_{-\pi}^{\pi} |\Delta_{y,\delta} B_{2i,2j}(x,y,t,s)|\, dy\right) dx \le C_{3,i,j}\tau(\delta);$$

– *for all* $0 \le i,j \le 1$ *exist constants* $M_{i,j} > 0$ *such that*

$$\left|\int_0^{\pi}\int_0^{\pi} \widetilde{B}_{2i,2j}(x,y,t,s)dxdy\right| \le \frac{M_{i,j}}{\varphi^2\left(\frac{1}{|t-s|}+v_0\right)}.$$

Then the series

$$u(x,t) = \sum_{n=1}^{\infty} X_n(x)\frac{1}{\mu_n}\int_0^t \sin\mu_n(t-u)\zeta_n(u)\, du, \tag{5}$$

where

$$\zeta_n(t) = \int_0^{\pi} \xi(x,t)X_n(x)\, dx,$$

uniformly converges in probability in the region $[0,\pi]\times[0,T]$ *(*$T>0$ *is some constant), the series, that obtained by diferentiation of the series* (5) *with respect to* x *one time and two times and with respect to* t *one time and two times, uniformly converge in probability and the problem* (1)–(3) *has a solution with probability one which is representable in the form of the series* (5).

Under the conditions of the theorem 1 we construct a model of solution of the problem (1)–(3) which approximate this solution with given reliability and accuracy in norm of the space $C([0,\pi]\times[0,T])$.

Let $\hat{\xi}(x,t)$ is a model of random field $\xi(x,t)$. Denote

$$\hat{\zeta}_n(t) = \int_0^{\pi} \hat{\xi}(x,t)X_n(x)dx.$$

The sum

$$\hat{u}^N(x,t) = \sum_{n=1}^{N} X_n(x)\frac{1}{\mu_n}\int_0^t \sin\mu_n(t-u)\hat{\zeta}_n(u)du \tag{6}$$

will be called the model of the process $u(x,t)$.

Definition 1. *The model $\hat{u}^N(x,t)$ approximate the solution of the problem* (1)–(3) $u(x,t)$ *which is representable in the form of the series* (5) *with given reliability* $1-\omega$ $(\omega \in [0,1])$ *and accuracy* $s \geq 0$ *in norm of the space* $C([0,\pi]\times[0,T])$ *if*

$$\mathsf{P}\left\{\sup_{\substack{x\in[0,\pi]\\ t\in[0,T]}} \left|\hat{u}^N(x,t) - u(x,t)\right| > s\right\} \leq \omega.$$

Denote

$$\Delta_N(x,t,N) = u(x,t) - \hat{u}^N(x,t) = u_N(x,t) + V_N(x,t),$$

where

$$u_N(x,t) = \sum_{n=N+1}^{\infty} X_n(x)\frac{1}{\mu_n}\int_0^t \sin\mu_n(t-u)\zeta_n(u)du,$$

$$V_N(x,t) = \sum_{n=1}^{N} X_n(x)\frac{1}{\mu_n} \times \int_0^t \sin\mu_n(t-u)\left(\zeta_n(u) - \hat{\zeta}_n(u)\right)du.$$

Theorem 2. *Let* $\xi(x,t)$ *is centered strong Orlicz sample continuous with probability one random field from space* $L_U(\Omega)$ *and for problem* (1)–(3) *the conditions of the theorem 1 hold true. We assume that the model* $\hat{\xi}(x,t)$ *such that*

$$\int_0^{\pi}\int_0^{T}\sqrt{\mathsf{E}\left(\xi(x,u) - \hat{\xi}(x,u)\right)^2}dudx < \chi.$$

Then random field $\hat{u}^N(x,t)$ *which determined in* (6) *is the model which approximate the random field* $u(x,t)$ *with reliability* $1-\omega$ *and accuracy* s *in norm of the space* $C([0,\pi]\times[0,T])$, *if* ω, s *and* N *such that for some* $\theta \in (0,1)$ *the next condition hold true*

$$\omega U\Bigg(s\theta(1-\theta)\Bigg(\int_0^{\theta w_0(N)} \chi_U\Bigg(\left(\frac{\pi}{2}\left(\varphi^{(-1)}\left(u^{-1}C_\Delta(\widehat{D}_N + \chi B_N)\right) - v_0\right) + 1\right)$$
$$\times\left(\frac{T}{2}\left(\varphi^{(-1)}\left(u^{-1}C_\Delta(\widehat{D}_N + \chi B_N)\right) - v_0\right) + 1\right)\Bigg)du\Bigg)^{-1}\Bigg) \geq 1,$$

where

$$w_0(N) = \frac{C_\Delta(\widehat{D}_N + \chi B_N)}{\varphi\left(\frac{1}{\max\{\pi,T\}} + v_0\right)},$$

$$\widehat{D}_N = T\max\{L, 2C_X\}\left(\sum_{k=N+1}^{\infty}\sum_{m=N+1}^{\infty}\frac{1}{\mu_k\mu_m}\widehat{C}_{k,m}\varphi(\mu_k^2+v_0)\varphi(\mu_m^2+v_0)\right)^{1/2}$$
$$+C_X\Bigg(\sum_{k=N+1}^{\infty}\sum_{m=N+1}^{\infty}\widehat{C}_{k,m}\frac{1}{\mu_k\mu_m}\big(4T^2\varphi(\mu_k+v_0)\varphi(\mu_m+v_0)$$
$$+2T\max\{1,2T\}\varphi(\mu_k+v_0)\varphi(1+v_0)$$
$$+2T\max\{1,2T\}\varphi(\mu_m+v_0)\varphi(1+v_0) + (\max\{1,2T\})^2\varphi^2(1+v_0)\big)\Bigg)^{1/2},$$

$$B_N = C_X \max\{L, 2C_X\} \sum_{n=1}^{N} \frac{\varphi(\mu_n^2 + v_0)}{\mu_n} + C_X^2 \left(\sum_{n=1}^{N} \sum_{k=1}^{N} \frac{1}{\mu_n \mu_k} \big(4\varphi(\mu_n + v_0)\varphi(\mu_k + v_0) \right.$$
$$+2\max\{1, 2T\}\varphi(1 + v_0)\varphi(\mu_n + v_0)\mu_k + 2\max\{1, 2T\}\varphi(1 + v_0)\mu_n\varphi(\mu_k + v_0)$$
$$\left. +(\max\{1, 2T\})^2\varphi^2(1 + v_0)\mu_n\mu_k\big) \right)^{1/2},$$

C_Δ is constant from the definition of strong Orlicz family of random variables [2],

$$\widehat{C}_{k,m} = \sum_{i,j=0}^{1} \frac{C_q^{2-i-j}}{\mu_k^2 \mu_m^2} \left(\frac{C_{1,i,j}\tau(\frac{\pi}{k}, \frac{\pi}{m})}{8\pi} + \sqrt{\frac{2}{\pi}} \frac{LC_{2,i,j}\tau(\frac{\pi}{k})}{4m} + \sqrt{\frac{2}{\pi}} \frac{LC_{3,i,j}\tau(\frac{\pi}{m})}{4k} + \frac{C_{G,i,j}}{km} \right),$$

$C_q = \sup_{0 \le x \le \pi} |q(x)|$, $C_{G,i,j} = L^2\pi^2 \max_{\substack{x\in[0,\pi]\\ y\in[0,\pi]\\ t\in[0,T]\\ s\in[0,T]}} |B_{2i,2j}(x,y,t,s)|$, $C_X = \sup_{\substack{x\in[0,\pi]\\ k\ge 1}} |X_k(x)|$,

L is constant from the paper [3] (such that $|X_k(x_1) - X_k(x_2)| \le L\mu_k^2|x_1 - x_2|,\ x_1, x_2 \in [0,\pi]$).

Proof. Since $\Delta_N(x,t)$ is a centered strong Orlicz random field from space $L_U(\Omega)$, according to theorem 1 from the paper [1] if exist continuous monotonously increasing function $\sigma(h), h \ge 0$ such that $\sigma(0) = 0$ and

$$\sup_{\substack{|x-y|\le h\\ |t-s|\le h}} \left(\mathsf{E}|\Delta_N(x,t) - \Delta_N(y,s)|^2\right)^{1/2} \le \frac{\sigma(h)}{C_\Delta},$$

and for some $\varepsilon > 0$ the following condition hold true

$$\int_0^{\varepsilon} U^{(-1)} \left(\left(\frac{\pi}{2\sigma^{(-1)}(u)} + 1 \right) \left(\frac{T}{2\sigma^{(-1)}(u)} + 1 \right) \right) du < \infty, \tag{7}$$

than for all $s > 0$ for all $0 < \theta < 1$

$$\mathsf{P}\left\{ \sup_{\substack{x\in[0,\pi]\\ t\in[0,T]}} |\Delta_N(x,t)| > s \right\} \le U^{-1}\left(\frac{s}{B(\theta)} \right),$$

where

$$B(\theta) = \frac{1}{\theta(1-\theta)} \int_0^{\theta w_0} \chi_U \left(\left(\frac{\pi}{2\sigma^{(-1)}(u)} + 1 \right) \left(\frac{T}{2\sigma^{(-1)}(u)} + 1 \right) \right) du,$$

$$w_0 = \sigma\left(\max\{\pi, T\}\right),\ \ \chi_U = \begin{cases} n, & n < U(z_0) \\ C_U U^{(-1)}(n), & n \ge U(z_0), \end{cases}$$

$C_U = K(1+U(z_0))\max\{1, A\}$, z_0, K, A are constants from definition of g-condition, $\sigma^{(-1)}(u)$ isthe inverse function of $\sigma(h)$.

It is easy to see that

$$\begin{aligned}&\sup_{\substack{|x-y|\le h\\|t-s|\le h}}\left(\mathsf{E}|\Delta_N(x,t)-\Delta_N(y,s)|^2\right)^{1/2}\\&=\sup_{\substack{|x-y|\le h\\|t-s|\le h}}\left(\mathsf{E}|u_N(x,t)+V_N(x,t)-u_N(y,s)-V_N(y,s)|^2\right)^{1/2}\\&\le\sup_{\substack{|x-y|\le h\\|t-s|\le h}}\left(\mathsf{E}|u_N(x,t)-u_N(y,s)|^2\right)^{1/2}+\sup_{\substack{|x-y|\le h\\|t-s|\le h}}\left(\mathsf{E}|V_N(x,t)-V_N(y,s)|^2\right)^{1/2}.\end{aligned}$$

According to lemma 1 and remark 1 from the paper [1]:

$$\sup_{\substack{|x-y|\le h\\|t-s|\le h}}(\mathsf{E}|u_N(x,t)-u_N(y,s)|^2)^{1/2}\le\frac{\widehat{D}_N}{\varphi\left(\frac{1}{h}+v_0\right)}.$$

$$\begin{aligned}&(\mathsf{E}|V_N(x,t)-V_N(y,s)|^2)^{1/2}\\&=\Bigg(\mathsf{E}\Bigg|\sum_{n=1}^N X_n(x)\frac{1}{\mu_n}\int_0^t\sin\mu_n(t-u)(\zeta_n(u)-\hat\zeta_n(u))du\\&-\sum_{n=1}^N X_n(y)\frac{1}{\mu_n}\int_0^s\sin\mu_n(s-u)(\zeta_n(u)-\hat\zeta_n(u))du\Bigg|^2\Bigg)^{1/2}\\&=\Bigg(\mathsf{E}\Bigg|\sum_{n=1}^N\frac{1}{\mu_n}(X_n(x)-X_n(y))\int_0^t(\zeta_n(u)-\hat\zeta_n(u))\sin\mu_n(t-u)du\\&+\sum_{n=1}^N\frac{1}{\mu_n}X_n(y)\Bigg(\int_0^t(\zeta_n(u)-\hat\zeta_n(u))\sin\mu_n(t-u)du\\&-\int_0^s(\zeta_n(u)-\hat\zeta_n(u))\sin\mu_n(s-u)du\Bigg)\Bigg|^2\Bigg)^{1/2}\le A_1+A_2,\end{aligned}$$

where

$$A_1=\left(\mathsf{E}\left|\sum_{n=1}^N\frac{1}{\mu_n}(X_n(x)-X_n(y))\int_0^t(\zeta_n(u)-\hat\zeta_n(u))\sin\mu_n(t-u)du\right|^2\right)^{1/2},$$

$$\begin{aligned}A_2=\Bigg(\mathsf{E}\Bigg|\sum_{n=1}^N\frac{1}{\mu_n}X_n(y)\Bigg(&\int_0^t(\zeta_n(u)-\hat\zeta_n(u))\sin\mu_n(t-u)du\\&-\int_0^s(\zeta_n(u)-\hat\zeta_n(u))\sin\mu_n(s-u)du\Bigg)\Bigg|^2\Bigg)^{1/2}.\end{aligned}$$

Denote

$$Q_{n,k}(t,s)=\int_0^t\int_0^s\sin\mu_n(t-u)\sin\mu_k(s-v)\mathsf{E}(\zeta_n(u)-\hat\zeta_n(u))(\zeta_k(v)-\hat\zeta_k(v))dvdu.$$

Then $A_1^2 \leq \sum_{n=1}^{N}\sum_{k=1}^{N}\frac{1}{\mu_n\mu_k}|X_n(x)-X_n(y)||X_k(x)-X_k(y)||Q_{n,k}(t,t)|$.

According to lemmas 2, 3 from the paper [3]

$$|X_n(x)-X_n(y)| \leq \max\{L,2C_X\}\frac{\varphi(\mu_n^2+v_0)}{\varphi\left(\frac{1}{|x-y|}+v_0\right)}.$$

Moreover

$$|Q_{n,k}(t,t)| \leq \int_0^T\int_0^T \left|\mathsf{E}(\zeta_n(u)-\hat{\zeta}_n(u))(\zeta_k(v)-\hat{\zeta}_k(v))\right| dudv$$
$$\leq \int_0^T \left(\mathsf{E}(\zeta_n(u)-\hat{\zeta}_n(u))^2\right)^{1/2} du \int_0^T \left(\mathsf{E}(\zeta_k(v)-\hat{\zeta}_k(v))^2\right)^{1/2} dv.$$

Then

$$\mathsf{E}(\zeta_n(u)-\hat{\zeta}_n(u))^2 = \mathsf{E}\left(\int_0^\pi X_n(x)(\xi(x,u)-\hat{\xi}(x,u))dx\right)^2$$
$$= \int_0^\pi\int_0^\pi X_n(x)X_n(y)\mathsf{E}(\xi(x,u)-\hat{\xi}(x,u))(\xi(y,u)-\hat{\xi}(y,u))dxdy$$
$$\leq C_X^2\left(\int_0^\pi \left(\mathsf{E}(\xi(x,u)-\hat{\xi}(x,u))^2\right)^{1/2}dx\right)^2.$$

We have

$$|Q_{n,k}(t,t)| \leq \left(C_X\int_0^\pi\int_0^T\sqrt{\mathsf{E}(\xi(x,u)-\hat{\xi}(x,u))^2}\,dudx\right)^2 \leq \chi^2 C_X^2.$$

Therefore

$$A_1^2 \leq \chi^2 C_X^2\left(\max\{L,2C_X\}\right)^2\sum_{n=1}^{N}\sum_{k=1}^{N}\frac{1}{\mu_n\mu_k}\varphi(\mu_n^2+v_0)\varphi(\mu_k^2+v_0)\left(\varphi\left(\frac{1}{|x-y|}+v_0\right)\right)^{-2}.$$

Namely

$$A_1 \leq \chi C_X\max\{L,2C_X\}\sum_{n=1}^{N}\frac{\varphi(\mu_n^2+v_0)}{\mu_n}\left(\varphi\left(\frac{1}{|x-y|}+v_0\right)\right)^{-1}.$$

$$A_2^2 \leq \sum_{n=1}^{N}\sum_{k=1}^{N}\frac{1}{\mu_n\mu_k}|X_n(y)X_k(y)|$$
$$\times\left|\mathsf{E}\left(\int_0^t(\zeta_n(u)-\hat{\zeta}_n(u))\sin\mu_n(t-u)du-\int_0^s(\zeta_n(u)-\hat{\zeta}_n(u))\sin\mu_n(s-u)du\right)\right.$$
$$\left.\times\left(\int_0^t(\zeta_k(v)-\hat{\zeta}_k(v))\sin\mu_k(t-v)dv-\int_0^s(\zeta_k(v)-\hat{\zeta}_k(v))\sin\mu_k(s-v)dv\right)\right|.$$

Let for certainty $s \le t$. Then

$$\int_0^t (\zeta_n(u) - \hat{\zeta}_n(u)) \sin \mu_n (t-u) du - \int_0^s (\zeta_n(u) - \hat{\zeta}_n(u)) \sin \mu_n (s-u) du$$
$$= \int_0^s (\zeta_n(u) - \hat{\zeta}_n(u)) \left(\sin \mu_n (t-u) - \sin \mu_n (s-u) \right) du + \int_s^t (\zeta_n(u) - \hat{\zeta}_n(u)) \sin \mu_n (t-u) du.$$

Therefore $A_2^2 \le C_X^2 \sum_{n=1}^{N} \sum_{k=1}^{N} \frac{1}{\mu_n \mu_k} q_{n,k}(t,s)$, where

$$q_{n,k}(t,s) = \bigg| \mathsf{E}\bigg(\int_0^s (\zeta_n(u) - \hat{\zeta}_n(u))(\sin \mu_n(t-u) - \sin \mu_n(s-u)) du$$
$$+ \int_s^t (\zeta_n(u) - \hat{\zeta}_n(u)) \sin \mu_n (t-u) du \bigg)$$
$$\times \bigg(\int_0^s (\zeta_k(v) - \hat{\zeta}_k(v))(\sin \mu_k(t-v) - \sin \mu_k(s-v)) dv$$
$$+ \int_s^t (\zeta_k(v) - \hat{\zeta}_k(v)) \sin \mu_k (t-v) dv \bigg) \bigg|$$
$$\le \int_0^s \int_0^s |\mathsf{E}(\zeta_n(u) - \hat{\zeta}_n(u))(\zeta_k(v) - \hat{\zeta}_k(v))| |\sin \mu_n(t-u) - \sin \mu_n(s-u)|$$
$$\times |\sin \mu_k(t-v) - \sin \mu_k(s-v)| dv du$$
$$+ \int_0^s \int_s^t |\mathsf{E}(\zeta_n(u) - \hat{\zeta}_n(u))(\zeta_k(v) - \hat{\zeta}_k(v))| |\sin \mu_n(t-u) - \sin \mu_n(s-u)|$$
$$\times |\sin \mu_k(t-v)| dv du$$
$$+ \int_s^t \int_0^s |\mathsf{E}(\zeta_n(u) - \hat{\zeta}_n(u))(\zeta_k(v) - \hat{\zeta}_k(v))| |\sin \mu_n(t-u)|$$
$$\times |\sin \mu_k(t-v) - \sin \mu_k(s-v)| dv du$$
$$+ \int_s^t \int_s^t |\mathsf{E}(\zeta_n(u) - \hat{\zeta}_n(u))(\zeta_k(v) - \hat{\zeta}_k(v))| |\sin \mu_n(t-u)| |\sin \mu_k(t-v)| dv du.$$

According to lemma 3 [3] ($Z_1(u) = u,\ u \in [0,T],\ \lambda = 1,\ C = 1,\ B = T$): for $u \in [s,t]$

$$|\sin \mu_n(t-u)| \le \mu_n |t-s| \le \max\{1, 2T\} \mu_n \frac{\varphi(1+v_0)}{\varphi\left(\frac{1}{|t-s|} + v_0\right)};$$

moreover ($Z_{\mu_n}(t) = \sin \mu_n (t-u),\ B = 1,\ \lambda = \mu_n,\ C = 1$)

$$|\sin \mu_n(t-u) - \sin \mu_n(s-u)| \le 2 \frac{\varphi(\mu_n + v_0)}{\varphi\left(\frac{1}{|t-s|} + v_0\right)}.$$

Then

$$q_{n,k}(t,s) \le \chi^2 C_X^2 \big(4\varphi(\mu_n + v_0)\varphi(\mu_k + v_0) + 2\max\{1,2T\}\varphi(\mu_n + v_0)\mu_k \varphi(1+v_0)$$
$$+ 2\max\{1,2T\}\mu_n \varphi(\mu_k + v_0)\varphi(1+v_0) + (\max\{1,2T\})^2 \mu_n \mu_k \varphi^2(1+v_0) \big) \frac{1}{\varphi^2\left(\frac{1}{|t-s|} + v_0\right)}.$$

It is clear that this inequality also holds for $t \le s$. So,

$$A_2 \le \chi C_X^2 \Bigg(\sum_{n=1}^{N} \sum_{k=1}^{N} \frac{1}{\mu_n \mu_k} \big(4\varphi(\mu_n + v_0)\varphi(\mu_k + v_0) + 2\max\{1, 2T\}\varphi(\mu_n + v_0)\mu_k\varphi(1 + v_0)$$

$$+2\max\{1, 2T\}\mu_n\varphi(\mu_k + v_0)\varphi(1 + v_0) + (\max\{1, 2T\})^2\mu_n\mu_k\varphi^2(1 + v_0)\big)\Bigg)^{1/2} \frac{1}{\varphi\left(\frac{1}{|t-s|} + v_0\right)}.$$

So

$$\sup_{\substack{|x-y|\le h\\ |t-s|\le h}} \left(\mathsf{E}|V_N(x,t) - V_N(y,s)|^2\right)^{1/2} \le \frac{\chi B_N}{\varphi\left(\frac{1}{h} + v_0\right)},$$

where

$$B_N = C_X \max\{L, 2C_X\} \sum_{n=1}^{N} \frac{\varphi(\mu_n^2 + v_0)}{\mu_n}$$

$$+C_X^2 \Bigg(\sum_{n=1}^{N} \sum_{k=1}^{N} \frac{1}{\mu_n \mu_k} \big(4\varphi(\mu_n + v_0)\varphi(\mu_k + v_0) + 2\max\{1, 2T\}\varphi(\mu_n + v_0)\mu_k\varphi(1 + v_0)$$

$$+2\max\{1, 2T\}\mu_n\varphi(\mu_k + v_0)\varphi(1 + v_0) + (\max\{1, 2T\})^2\mu_n\mu_k\varphi^2(1 + v_0)\big)\Bigg)^{1/2}.$$

Finally

$$\sup_{\substack{|x-y|\le h\\ |t-s|\le h}} \left(\mathsf{E}|\Delta_N(x,t) - \Delta_N(y,s)|^2\right)^{1/2} \le \frac{\widehat{D}_N + \chi B_N}{\varphi\left(\frac{1}{h} + v_0\right)},$$

namely

$$\sup_{\substack{|x-y|\le h\\ |t-s|\le h}} \left(\mathsf{E}|\Delta_N(x,t) - \Delta_N(y,s)|^2\right)^{1/2} \le \frac{\sigma(h)}{C_\Delta},$$

where

$$\sigma(h) = \frac{C_\Delta(\widehat{D}_N + \chi B_N)}{\varphi\left(\frac{1}{h} + v_0\right)}, \;\; h > 0.$$

Then

$$\sigma^{(-1)}(u) = \frac{1}{\varphi^{(-1)}\left(\frac{C_\Delta(\widehat{D}_N + \chi B_N)}{u}\right) - v_0}, \;\; u > 0,$$

$$w_0 = \sigma\left(\max\{\pi, T\}\right) = \frac{C_\Delta(\widehat{D}_N + \chi B_N)}{\varphi\left(\frac{1}{\max\{\pi, T\}} + v_0\right)} = w_0(N),$$

$$B(\theta) = \frac{1}{\theta(1-\theta)} \int_0^{\theta w_0(N)} \chi_U \Bigg(\left(\frac{\pi}{2}\left(\varphi^{(-1)}\left(u^{-1}C_\Delta(\widehat{D}_N + \chi B_N)\right) - v_0\right) + 1\right)$$

$$\times \left(\frac{T}{2}\left(\varphi^{(-1)}\left(u^{-1}C_\Delta(\widehat{D}_N + \chi B_N)\right) - v_0\right) + 1\right) \Bigg) du,$$

condition (7) is fulfilled because the condition (4) is fulfilled.

Therefore, in order to obtain a solution model with $1-\omega$ reliability at s accuracy, it is necessary that for some $\theta \in (0,1)$ the condition

$$U^{-1}\Bigg(s\theta(1-\theta)\Bigg(\int_0^{\theta w_0(N)} \chi_U\Bigg(\left(\frac{\pi}{2}\left(\varphi^{(-1)}\left(u^{-1}C_\Delta(\widehat{D}_N+\chi B_N)\right)-v_0\right)+1\right) \times \left(\frac{T}{2}\left(\varphi^{(-1)}\left(u^{-1}C_\Delta(\widehat{D}_N+\chi B_N)\right)-v_0\right)+1\right)\Bigg)du\Bigg)^{-1}\Bigg) \le \omega$$

was fulfilled. □

References

[1] Dovhai B.V. *The estimation of supremum distribution of solution of hyperbolic equations with Orlicz right side* Carpathian Mathematical Publications 2013,**5** (2), 231—241. (in Ukrainian)

[2] Dovhai B.V. *The equations of string oscillation with Orlicz right part* Scientific Bulletin of the Uzhgorod University. Series Mathematics and Informatics 2013, **24** (1), 34–45. (in Ukrainian)

[3] Dovhai B.V. *Hyperbolic equations with Orlicz right-hand side* Scientific Bulletin of the Uzhgorod University. Series Mathematics and Informatics 2011, **22** (2), 64–78. (in Ukrainian)

[4] Dovhai B.V. *Simulation of solution of hyperbolic equations* Bulletin of Kyiv National Taras Shevchenko University. Series: Physics & Mathematics 2011, (3), 18–23. (in Ukrainian)

[5] Polozhii G.N. Equations of mathematical physics. Vysshaya Shkola, Moscow, 1964. (in Russian)

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Робота присвячена побудові моделі розв'зку гіперболічного рівняння з випадковою строго Орлічевою правою частиною та нульовими початковими і крайовими умовами, яка наближає цей розв'язок із заданою точністю та надійністю в рівномірній нормі.