Random walks in a strongly sparse random environment

Dariusz Buraczewski
University of Wrocław

March 17th, 2022

50th anniversary of Alexander Iksanov

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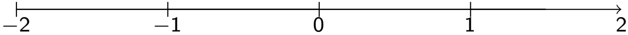

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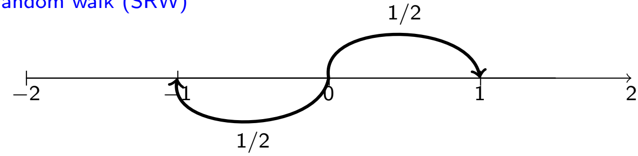

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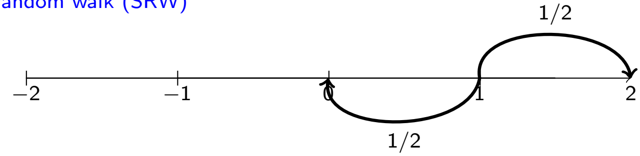

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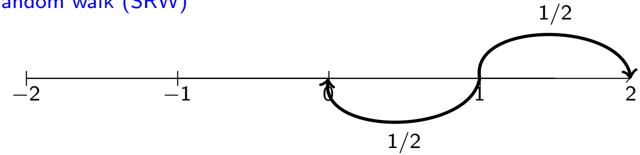

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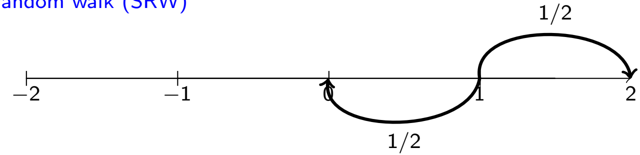

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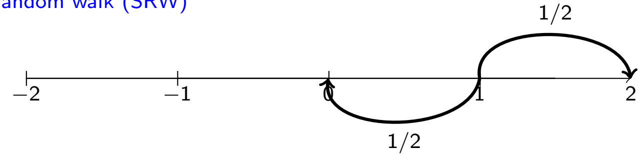

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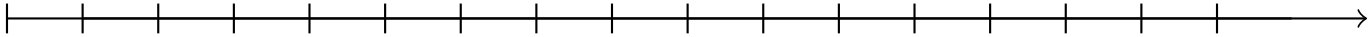

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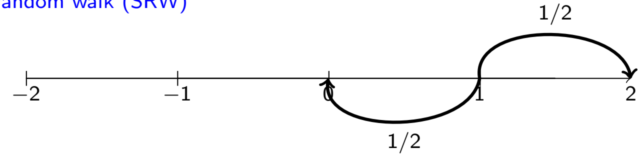

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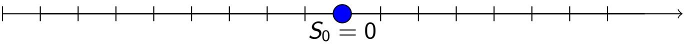

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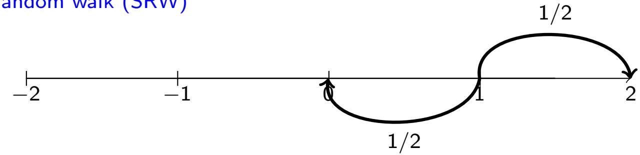

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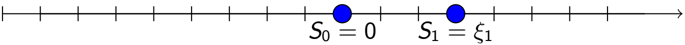

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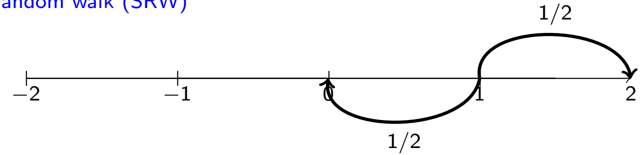

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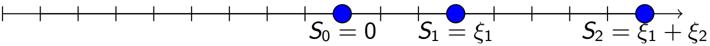

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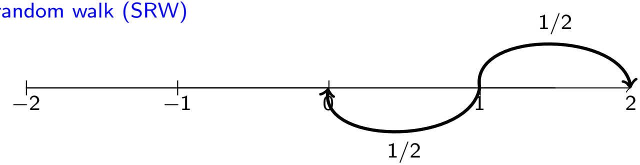

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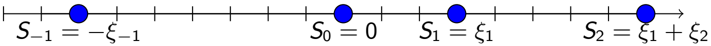

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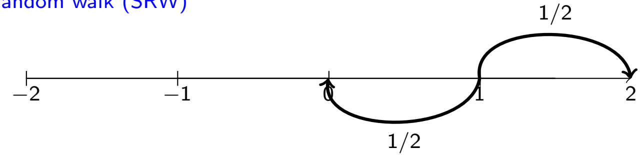

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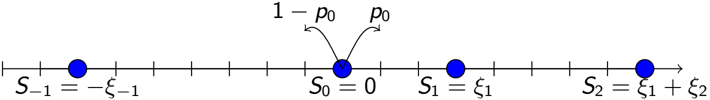

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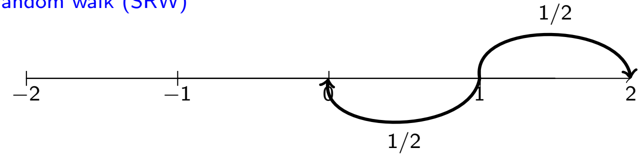

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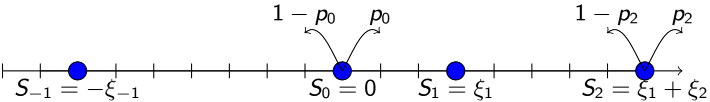

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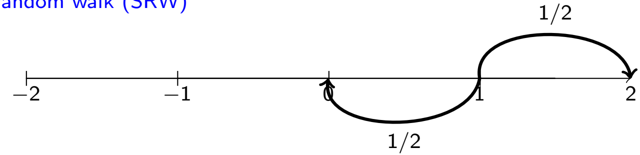

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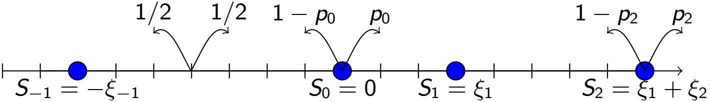

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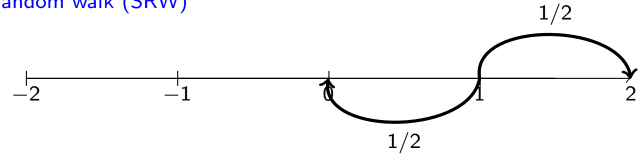

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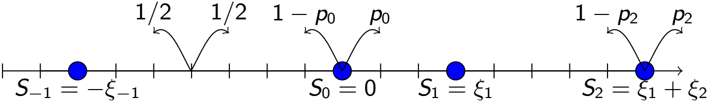

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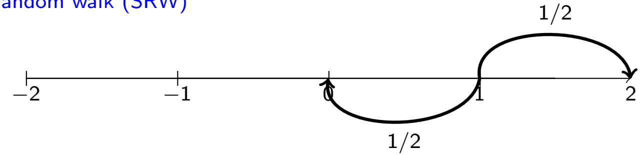

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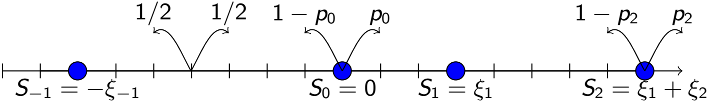

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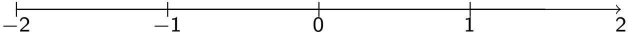

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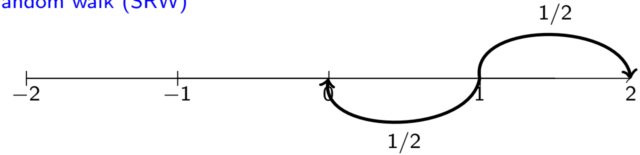

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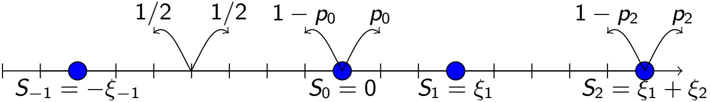

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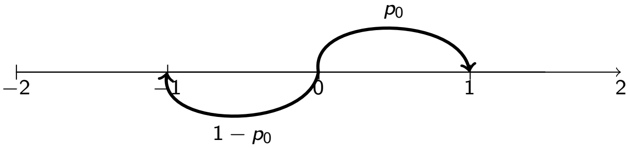

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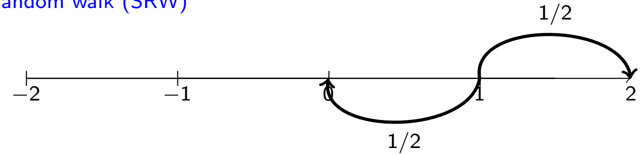

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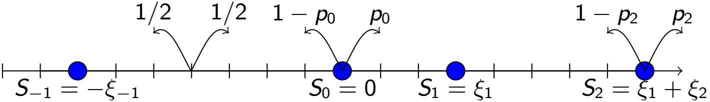

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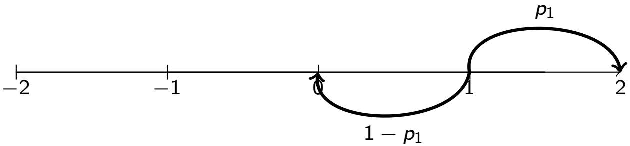

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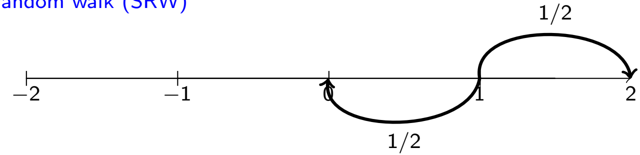

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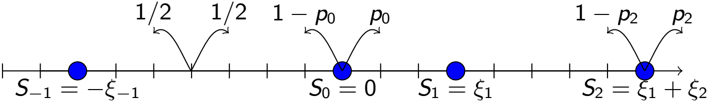

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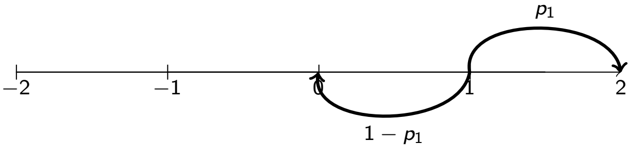

$$\mathbb{P}[X_{n+1} = k+1 \,|X_n = k] = p_k, \quad \mathbb{P}[X_{n+1} = k-1 \,|X_n = k] = 1 - p_k.$$

RWRE

Let $\rho = \frac{1-p}{p}$.

Recurrence and transience (Solomon, 75):

- If $\mathbb{E} \log \rho = 0$, then $\liminf X_n = -\infty$, $\limsup X_n = \infty$, $\mathbb{P}$ a.s.
- If $\mathbb{E} \log \rho < 0$, then $X_n \to \infty$, $\mathbb{P}$ a.s.

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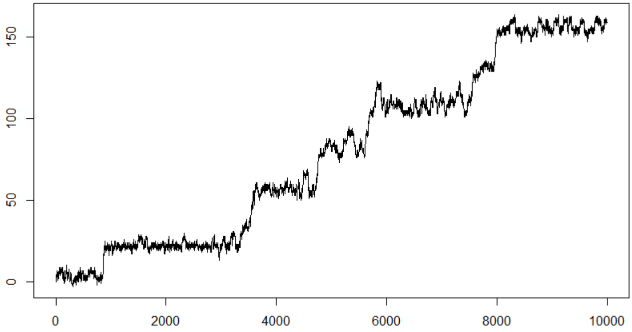

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Theorem (Solomon '75, Law of large numbers)

If $\mathbb{E} \log \rho < 0$

$$\lim_{n \to \infty} \frac{X_n}{n} = v, \quad \mathbb{P} \text{ a.s. and } v > 0 \text{ iff } \mathbb{E}\rho < 1.$$

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Theorem (Kesten, Kozlov, Spitzer '75, Central Limit Theorem)

Assume RWRE is transient ($\mathbb{E}[\log \rho] < 0$) and ... Then

$$\frac{X_n - vn}{a_n} \Rightarrow \mathcal{L}_\alpha.$$

If $\mathbb{E}\rho^2 < 1$, then $a_n = \sqrt{n}$ and $\mathcal{L}_\alpha = N(1, \sigma^2)$. Otherwise the limit and normalization are related to the parameter α such that $\mathbb{E}[\rho^\alpha] = 1$

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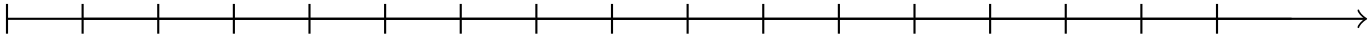

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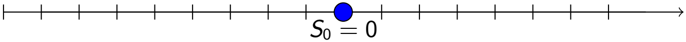

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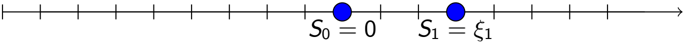

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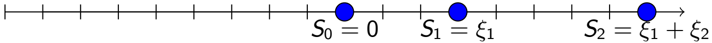

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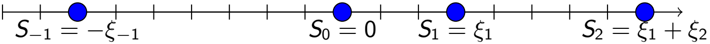

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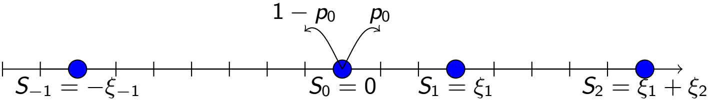

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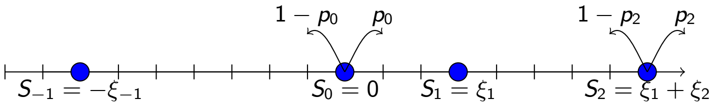

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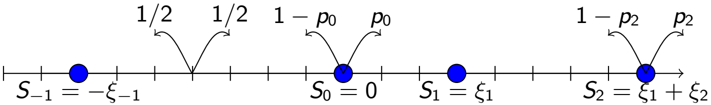

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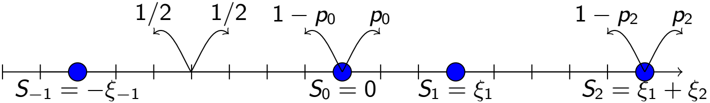

Lemma: If $\mathbb{E} \log \rho < 0$ (for $\rho = \frac{1-p}{p}$) and $\mathbb{E} \log \xi < \infty$, then $X_n \to +\infty$.

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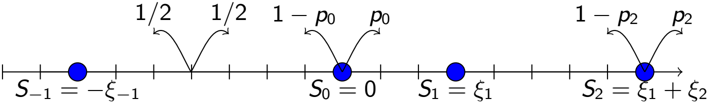

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Theorem (B, Dyszewski, Iksanov, Marynych, Roitershtein)

I. Assume $\mathbb{E}\xi < \infty$.

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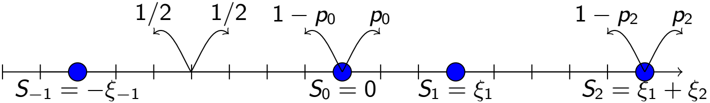

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Random walk in sparse random environment (RWSRE)

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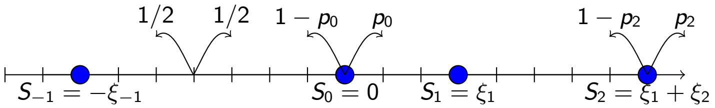

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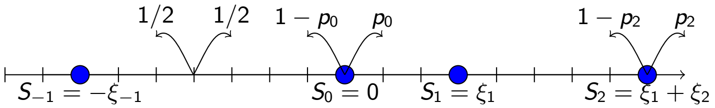

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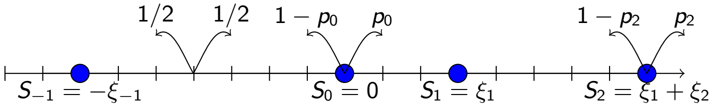

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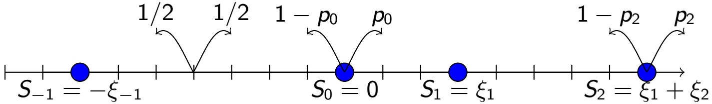

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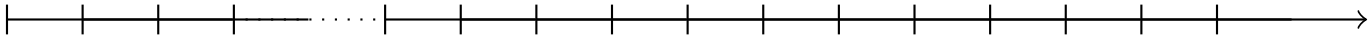

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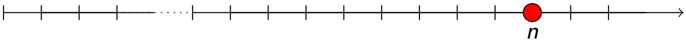

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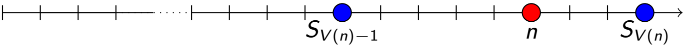

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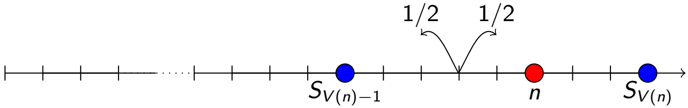

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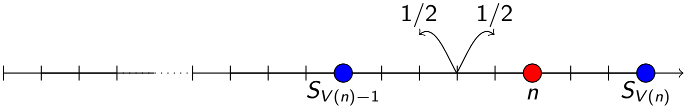

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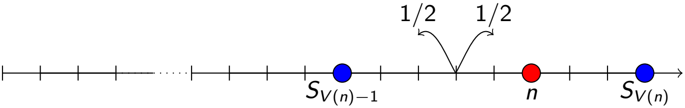

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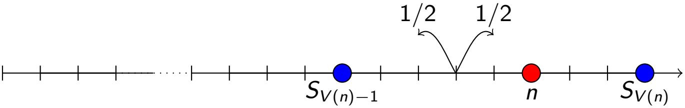

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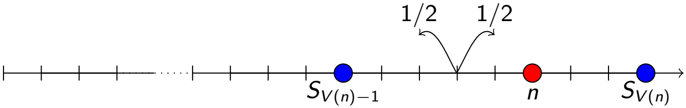

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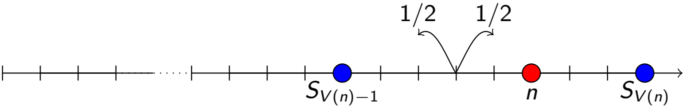

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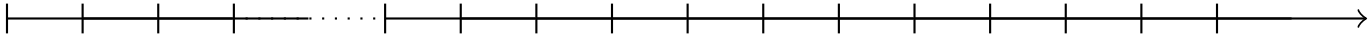

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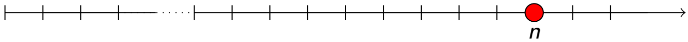

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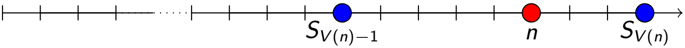

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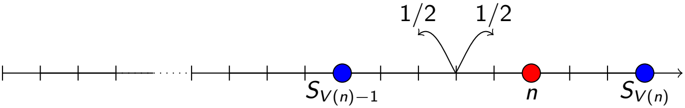

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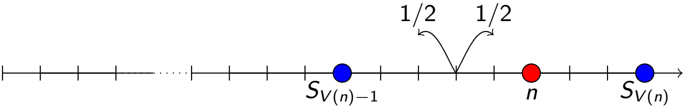

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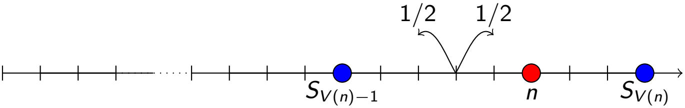

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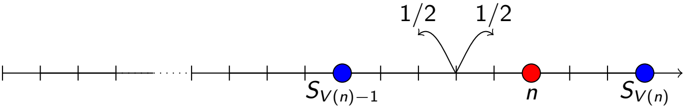

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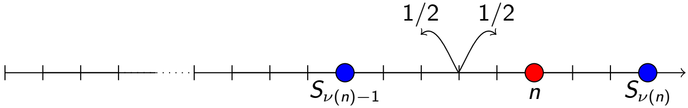

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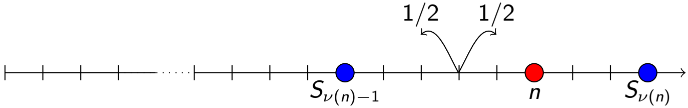

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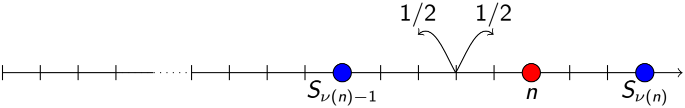

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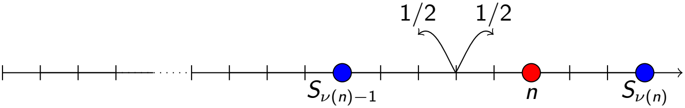

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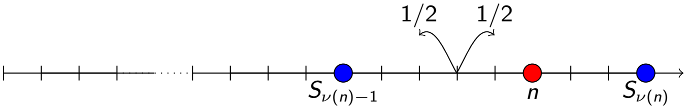

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Hence

$$\frac{T_n}{n^2} \to \chi_0$$

$$\frac{X_n}{\sqrt{n}} \to \chi_0^{-1/2}.$$

To describe ξ_0

- need to understand joint law of (ξ, T'_ξ)
- use branching processes
- ...

Thank you for your attention


Слава Україні!
Героям Слава!